Valuing a Lost Opportunity: An Alternative Perspective on the Illiquidity Discount

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Executive Summary

- As private strategies play a more prominent role in investors' portfolios, many asset allocators are interested in what *illiquidity discount* they should command as compensation for tying up capital.
- When investors commit capital to a fund with lockups, they effectively give up the opportunity to take advantage of future opportunities.
- We take an alternative approach to assessing the illiquidity discount by modeling this *opportunity cost*.
- The "optimal" illiquidity discount will compensate the investor for this opportunity cost.
- The illiquidity discount investors should command will reflect their perceived skill and alpha-generating abilities.
- We find illiquidity discounts on the order of 1.5%–2% per year are reasonable for most investors, but they are much higher for investors who are highly skilled.

Vehicles that hold illiquid investments such as private equity, private credit or real estate often require that end investors maintain their capital investments in the vehicles for a minimum period of time – three to five years or even longer. This capital lockup should, in principle, force investors to command higher expected returns. Accordingly, investors set their reservation price for shares at a level below what they would command if the underlying investments were perfectly liquid. This price gap is called the *illiquidity discount*. The increased expected return the investor earns as a result of paying a lower price for less liquid investments is called the *illiquidity premium*.

The illiquidity premium has been a topic of consideration since John Maynard Keynes wrote that one of the motives for holding liquidity (cash) is for the purpose of future speculation. As Keynes stated in his 1936 book, *The General Theory of Employment, Interest and Money*, "The rate of interest at any time, being the reward for parting with liquidity, is a measure of the unwillingness of those who possess money to part with their liquid control over it." In more recent work, researchers have discussed several reasons for the existence of the illiquidity premium. The primary justifications fall into four broad categories: clientele effects, time-varying transaction costs, portfolio inefficiency and opportunity cost.

Amihud and Mendelson (1986) show that when investors have differing needs for liquidity (known as clientele effects), this leads to their "specializing" in holding the securities that align most closely with their liquidity horizons. The authors conclude that patient investors are able to earn an illiquidity premium relative to their less patient counterparts. Acharya and Pedersen (2005) and Ang et al. (2014) note that markets are subject to periods in which liquidity dries up and investors are unable to liquidate positions (or must do so at onerous prices); in markets characterized by time-varying liquidity, less liquid assets earn a risk premium as compensation for future liquidity uncertainty. Longstaff (2017) uses an option pricing framework to show that investors holding illiquid assets should be compensated for the opportunity cost of being precluded from selling the assets at a favorable valuation over the investment horizon. In this framework, omniscient investors who hold perfectly illiquid assets are effectively short a lookback option on the underlying investment and hence should earn an illiquidity premium for exposing themselves to this particular source of risk.

In this paper, we posit a formal framework for quantifying the illiquidity discount through the lens of the opportunity cost. In our model, the opportunity cost stems from the investor's expectations of "available alpha" in the market. Available alpha is assumed to come from two sources: 1) a continuous alpha stream that the investor can earn from exploiting relatively known alpha sources, such as selection and timing, and 2) an alpha jump component, which infrequently materializes but can result in significant buying opportunities. The 2008 credit crisis is an example of the jump component. Alpha is a particularly convenient measure of opportunity cost because it doesn't require estimates of risk premia; investors discount future values of alpha at the risk-free rate instead of highly subjective and asset-specific discount rates.

Hence, the illiquidity discount that investors should command is a function of how they perceive the alpha landscape over the capital commitment horizon. If they view the opportunity as significant, the cost of tying up capital is relatively high and they should command a larger illiquidity discount. Conversely, if they perceive the opportunity as scarce or unlikely to emerge over the investment horizon, investors should be more comfortable paying a price for an illiquid asset that is more in line with its fully liquid market valuation. In markets where "normal" alpha opportunities are expected and the likelihood of a crisis is near its unconditional probability, the illiquidity discount should be somewhere between these two extremes. Our empirical results indicate that for reasonably capable investors, the illiquidity discount ranges around 1.5% to 1.9% per year. For highly skilled investors, however, the discount could range from 4.1% to 7.2% per year.

1. METHODOLOGY

Our framework is based on the simple fact that if an asset is liquid in the sense that it can be easily traded for minimal cost, the investor can access his capital at will to take advantage of future opportunities. This could include dynamically shifting asset selection, timing the market or taking advantage of large future dislocations. In contrast, investors who commit capital to illiquid investments do not have such flexibility and therefore should command return compensation in the form of an illiquidity discount.[1] Investors are assumed to be unable to borrow against the illiquid asset. If an investor could do so, he would be able to effectively use his capital to exploit the same opportunities as if he were in a fully liquid position.[2] In our framework, an illiquidity discount would not be economically justified in such a scenario.

We model the forgone alpha as coming from two distinct sources: a continuous alpha process and a jump process. The continuous process is employed to model general opportunities through day-to-day alpha-seeking efforts such as timing, asset selection and position sizing, whereas the jump process is used to mimic a "dry powder" strategy that buys

1 Although there is likely some price at which an investor could sell an illiquid asset should he need to, we are making the assumption that the transaction, operational and legal costs associated with doing so are sufficiently prohibitive that an investor would not be willing to incur them in any circumstance.

2 This same assumption is made in Ang et al. (2014) and Amihud and Mendelson (1986), in which the investor is constrained from borrowing against his illiquid investment for purposes of smoothing consumption. For a general discussion of the relationship between borrowing frictions and market liquidity, see Brunnermeier and Pedersen (2009).

cheap assets whenever markets experience a significant sell-off. We model the continuous alpha opportunity, α, as an Ornstein–Uhlenbeck (OU) process (Vasicek 1977):

$$d\alpha_t = k(\mu - \alpha_t)dt + \sigma dW_t \quad (1)$$

where $k > 0$ represents the mean-reverting speed, μ is the long-term mean of available alpha in the market, $\sigma > 0$ is the instantaneous alpha volatility of $d\alpha$ and W_t denotes the standard Wiener process.

The jump component, on the other hand, is designed to capture infrequent but substantial opportunities expected to arise in the future. It is modeled as a Poisson counting process q_t. Given its intensity, λ, the probability of an instantaneous jump is $P\ dq_t \geq 1\ = \lambda dt$. Hence, the instantaneous expected return on a dry-powder strategy is

$$E(Jdq_t) = \lambda J dt \quad (2)$$

where E is the standard expectation operator and J is the alpha an investor earns in a crisis period. In other words, the expected profit from jumps is equal to the probability of a jump occurring (λdt) multiplied by the payoff in crisis states (J).

The value of the total alpha opportunity, V, over the lockup period from time 0 to T is the expected present value of the discounted opportunity, accruing from both the continuous and the jump components:

$$V = E\left(\int_0^T e^{-rt}(\alpha_t dt + J dq_t)\right) \quad (3)$$

where r is the risk-free interest rate.[3] Combining Equations 1 and 2, Equation 3 can be solved as (see appendix for details):

$$\mathrm{V} = \frac{\mu}{r}(1 - e^{-rT}) + \frac{\lambda J}{r}(1 - e^{-rT}) + \frac{\alpha_0 - \mu}{r+k}\left(1 - e^{-(r+k)T}\right), \quad (4)$$

where α_0 represents the alpha state at the beginning of the period.

The first term in Equation 4 represents the capitalized value of the continuous alpha stream, adjusted for the time horizon of the capital lockup. Shorter time horizons imply lower realizable alpha value, all else equal. Note that as $T \to \infty$, the first term is simply the perpetuity value of realizable alpha. The second term reflects the capitalized value of dry powder. Intuitively, when the long-term alpha opportunity μ is high, crises are frequent in terms of large λ; or when the payoff in crises, J, is large, the alpha value is high. The last term in Equation 4 represents the current value of realizable alpha relative to its long-term mean. When we perceive the immediate alpha opportunity as being above its long-run value, this has a positive impact on the value of realizable alpha. Note that unlike the first two terms in Equation 4, the last term includes the mean-reversion speed, k, as a discount parameter. If $\alpha_0 > \mu$, then a faster mean-reversion speed decreases the value of realizable alpha, as a higher k implies a shorter window of time over which to earn the excess alpha. Unless one has strong prior knowledge on the value of α_0, it may be reasonable to assume $\alpha_0 = \mu$, making only the first two terms in the equation relevant (For a further discussion on this topic see Shimko, 1992).

Equation 4 is the present value of the forgone alpha opportunity — that is, the liquidity premium. We convert V into an illiquidity discount, Δ, in the following way:

$$\Delta = \frac{\mathrm{V}}{1+\mathrm{V}} \quad (5)$$

Thus, Δ is the discount that an investor would apply to a nonmarketable investment relative to its fully marketable value. By commanding an illiquidity discount, the investor is effectively increasing the required return of the illiquid asset so as to earn both its liquid-equivalent return and the forgone alpha V. Equation 5 has the desirable property of being bounded by 100%.[4]

Investors with varied experience and confidence in their alpha-seeking abilities could come up with their own specifications for Equation 4. A skillful investor such as Warren Buffett, who has shown the repeated ability to invest significantly (and profitably) when markets are in turmoil, would view liquidity as much more valuable than a less skilled investor would. In the context of Equation 4, this implies that $J^{Buffett} > J^{Average}$ or $\mu^{Buffett} > \mu^{Average}$. Ultimately, the illiquidity discount will be unique to each investor and will be based on their subjective view of the relevant model parameters. In the next section, we attempt to parameterize the model for a typical investor by estimating sensible parameter values based on the historical track record of realizable alpha.

2. MODEL ESTIMATION

In this section, we put forth what we believe to be reasonable parameter values for Equation 4. Our intention is to provide sensible estimates of the alpha opportunity rather than

3 We discount at r rather than at some assumed risk premium because the opportunity is modeled as coming from alpha, as opposed to beta.

4 Although it may seem intuitive that the illiquidity discount would have a lower bound of zero, that doesn't have to be the case within the context of our model. If, for example, the investor perceived the value of future alpha to be negative, then he would effectively pay an illiquidity premium (a negative discount) to avoid investing in negative alpha opportunities.

advocating these values as the "correct" model inputs. To the extent that investors have different views from what we estimate here, they should feel free to use them and come up with their own illiquidity discounts.

To estimate the parameters of the continuous alpha process, we create a simple representative trading strategy and estimate its alpha. Specifically, we take a universe of 35 equity index futures around the globe and rank them by their index dividend yield. We then form monthly portfolios that long the top three dividend-yielding futures and short the bottom three. Each position is held inversely proportional to its trailing 36-month volatility, estimated using monthly data. Leverage is scaled to target 10% ex ante volatility. We assume a modest transaction cost of 5 basis points each way. Because we model the time dependency of alpha vis-à-vis a mean-reversion process, we run rolling 24-month regressions of the strategy return on the MSCI ACWI index net of three-month Libor. Using the maximum likelihood estimator (Berg 2011) allows us to estimate all of the relevant parameters of the continuous alpha process.

The parameters of the jump component are determined through a simple dry-powder strategy that attempts to determine highly attractive buying opportunities during stressed periods. To assess the size of such opportunities, we monitor the S&P 500 index for periods in which its peak-to-trough drawdown exceeds the annualized volatility inferred from the past 36 monthly returns; in these cases, a stress event is triggered. When such a stress event occurs, we look to a universe of 11 equity, fixed income and commodity futures and buy those that have experienced peak-to-trough drawdowns in excess of 2 standard deviations. We hold each investment for 24 months, at which point it is automatically sold. Though the thresholds and rules we use are admittedly subjective, our intention is merely to put forth a strategy that mines for bargains. Exhibit 1 shows the parameter estimates for the continuous and jump alpha components.

Based on the results in Exhibit 1, we set the following annualized parameter values for each component of the model: μ = 1.0%, J = 5%, λ = 0.2, and k = 1.05. Investors would want to use the prevailing level of interest rates when assessing the illiquidity discount. For simplicity, we assume that the current level of alpha, α_0, is equal to its long-run level of 1.0%.

The parameter values in Exhibit 1 are designed to capture the alpha potential of investors with a reasonable level of skill. This means investors who possess superior selection and timing, as well as dexterity and willingness to purchase distressed assets when valuations are favorable. Skill is, of course, a continuum, with investors on both sides of the distribution. To illustrate our model predictions for particularly skilled investors, we use the example of Warren Buffett, who has shown a consistent ability to add alpha throughout his long and storied career. Frazzini et al. (2018) estimate Buffett's information ratio from 1976 to 2017 to be an impressive 0.79. As we do in the case of the continuous alpha component, we scale this value to a constant 10% volatility strategy, yielding an alpha for Buffett of 7.9%. We use this value in Equation 4 to show how the illiquidity discount would change for an investor who possesses an unusually high level of skill.

Exhibit 1: Estimation of model parameters

Continuous process	μ	0.010
	k	1.050
	σ	0.042
Jump process	J	0.05
	λ	0.200

Source: PIMCO and Bloomberg as of 31 March 2018

Exhibit 2: Total and annual illiquidity discounts versus the investment horizon

		Investment horizon (in years)									
	Skill level	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Total discount	Average	1.9%	3.8%	5.5%	7.1%	8.6%	10.0%	11.3%	12.6%	13.8%	15.0%
	Buffett	7.2%	13.3%	18.6%	23.1%	27.0%	30.5%	33.6%	36.3%	38.8%	41.0%
Annual discount	Average	**1.9%**	1.9%	**1.9%**	1.8%	1.8%	1.7%	1.7%	1.7%	1.6%	1.6%
	Buffett	7.2%	6.9%	6.6%	6.4%	6.1%	5.9%	5.7%	5.5%	5.3%	5.1%

Source: PIMCO and Bloomberg as of 31 March 2018

Exhibit 2 shows the illiquidity discounts from Equation 5 for both the average and the highly skilled (Buffett) investor versus the investment horizon. For example, using the parameter values from Exhibit 1 and an interest rate of 2.6% (the five-year U.S. government bond yield as of this writing), we find that the illiquidity discount associated with a five-year illiquidity horizon is 8.6%, or 1.8% per year for the average investor.[5] The illiquidity discount ranges from 1.9% for a single-year lockup to 15.0% (1.6% per year) for a 10-year lockup.[6] For highly skilled, Buffett-type investors, discounts are more than double those of the average investor, ranging from 7.2% for a one-year investment horizon to 41.0% (5.1% per year) for a 10-year horizon. This difference reflects the important role that an investor's skill plays in determining his illiquidity discount, with highly skilled investors requiring a much larger return compensation for committing their capital.

3. AN ALTERNATIVE BEHAVIORIAL ANGLE

In the previous section, we posited a model in which the illiquidity discount is "rationally" priced based on the alpha-generating abilities of the investor. Based on simple representative trading strategies designed to achieve a reasonable level of alpha, we found that investors should command illiquidity discounts on the order of 1.6% to 1.9% per year. In this section, we assert that the illiquidity discount need not be based on what is attainable but rather should be a function of the investor's *perceived* skill.

Asset prices are based on the beliefs of the marginal investor. If the marginal investor believes the asset's value to be higher (lower) than the market price, the price will rise (fall) until it fully reflects their views. Equation 4 shows that the value of liquidity (or, equivalently, the cost of illiquidity) is related to the alpha opportunity over the capital commitment period. As such, the illiquidity discount at any given time will represent the marginal investor's perception of his alpha-generating capabilities. The term "perception" is important. It implies that the investor's actual alpha-generating capabilities are irrelevant; what matters is what the investor believes he can achieve, whether or not the belief is justified. To the extent, for example, that the marginal investor believes he has greater alpha-generation skill than he actually does, the market-clearing illiquidity discount will be too high compared with the true alpha that can be achieved. This introduces an interesting behavioral angle to our model in which the market-clearing illiquidity discount should be considered not on the basis of what is realistic – efficient market advocates view alpha as zero, on average — but rather on the basis of investors' views of their own abilities.

In this context, there is evidence to suggest that individuals may overestimate their mental aptitude when it comes to assessing their own cognitive abilities. Several studies have found that males in particular tend to overestimate their IQs, with findings of overestimation ranging between 2.5 and 7.8 IQ points (Reilly and Mulhern 1995).[7] If, indeed, this overconfidence extends to the pricing of financial assets–the illiquidity discount in particular–this implies that within the context of Equation 4 illiquidity discounts may be higher than necessary, reflecting biased self-perceptions of alpha skill. Interestingly, translating IQ overestimation into the equivalent of a Sharpe ratio, by dividing by the IQ standard deviation of 15, implies a Sharpe ratio of between 0.17 and 0.52. These values lead to results not dissimilar from those found in the previous section.

4. DISCUSSION

Absent the opportunity cost argument, investors who have structured their allocations to illiquid assets in such a way that their liquidity needs are not unduly burdened have little justification for commanding large illiquidity discounts. Traditional arguments around portfolio inefficiency stemming from the inability to rebalance to restrictions on consumption smoothing are unlikely to be material as long as investors have not committed their portfolios to the point that liquidity is likely to be an issue in the future. This leaves opportunity cost as the primary justification for illiquidity discounts. To the extent that the investor believes there are likely to be future opportunities in which excess profits can be earned, the opportunity cost of committing capital to an illiquid investment may be high, so the illiquidity discount should be high as well.

Within the context of Equation 4, the term α_0 can be useful for understanding the potential dynamics of the illiquidity discount with respect to the business cycle. The term α_0 measures the

5 The annualized illiquidity discount is equal to $\Delta^{ann} = \left[(1+V)^{1/T} - 1\right]/(1+V)^{1/T}$.

6 The astute reader may recognize that for a given set of model parameters, the per year illiquidity discount decreases with time (although the total illiquidity discount increases with time). This occurs primarily due to the effect of discounting the future alpha opportunities at the interest rate, r. Because the term $1 - e^{-rT}$ in Equation 4 increases at a decreasing rate over time, this implies that the value of liquidity will be concave with respect to T. In the case where r = 0, for example, the value of liquidity will increase linearly with time and the per period value of liquidity will be a constant.

7 For additional findings on the relationship between gender and self-reported IQ, see Furnham and Rawles (1995), Furnham and Rawles (1999) and Furnham, Reeves and Budhani (2001).

current alpha opportunity and becomes a relevant input only when it deviates from the long-run alpha term, μ. When this deviation is positive, it implies that the current alpha landscape is above average. Conversely, when the term is negative, alpha opportunities are perceived to be relatively scarce. Although we haven't explicitly calibrated this parameter in this paper, it would be reasonable to conjecture that α_0 should behave countercyclically, increasing when the economy is in a bad state and decreasing in good times. When the economy is doing poorly, such as during a recession, the opportunity to earn abnormal returns is arguably more pronounced, implying a high level of α_0. Conversely, in good times alpha opportunities are likely scarce, meaning that α_0 should be below average. Equation 5 implies that the illiquidity discount should be highest during recessions and lowest in the latter part of business-cycle expansions.

In fact, countercyclical illiquidity discounts are generally confirmed in the empirical data. Nadauld et al. (2018) find that the discount to fair value for private funds traded in the secondary market is highest when the economy is doing poorly and lowest when it is doing well. For example, during the 2006–2014 sample period, the authors found average illiquidity discounts of 45.6% and 6.8% in 2009 and 2014, respectively. These periods include extremes in the global economy, with 2009 being the depth of the credit crisis and 2014 representing the heart of the post-2009 expansion. To highlight this concept, Exhibit 3 shows the impact on the total and annual illiquidity discount as a function of α_0, assuming the same parameter values as in Exhibit 1 and a five-year investment horizon.[8] When the current alpha opportunity is equal to the long-run alpha of 1%, the annual illiquidity discount is the same as in Exhibit 2a: 1. 8%. However, the annual discount ranges from 1.3% when alpha is scarce at -2% to 2.3% when the current alpha is 4%.

Exhibit 3: Total and annual illiquidity discounts versus alpha opportunity (α_0)

	Current alpha opportunity						
	-2%	-1%	0%	1%	2%	3%	4%
Total discount	6.2%	7.0%	7.8%	8.6%	9.3%	10.1%	10.8%
Annual discount	1.3%	1.4%	1.6%	1.8%	1.9%	2.1%	2.3%

Source: PIMCO and Bloomberg as of 31 March 2018

5. CONCLUSION

A major justification for the illiquidity discount is the opportunity cost of being unable to exploit future opportunities. In essence, illiquid assets must compensate their holders for their inability to earn excess returns through alpha-seeking behavior. To the extent that investors believe the road to future wealth is paved with opportunities, they should command higher illiquidity discounts. Doing so effectively increases required return on the illiquid asset to compensate for a perceived future windfall. Investors should be cognizant that the illiquidity discount behaves countercyclically: It is highest during recessions and lowest in late expansions. Within the context of our model, this implies that investors perceive alpha opportunities to be greatest at the depth of business-cycle recessions.

REFERENCES

Acharya, Viral V. and Lasse Heje Pedersen. "Asset Pricing with Liquidity Risk." *Journal of Financial Economics*, August 2005

Amihud, Yakov and Haim Mendelson. "Asset Pricing and the Bid-Ask Spread." *Journal of Financial Economics*, December 1986

Ang, Andrew, Dimitris Papanikolaou and Mark M. Westerfield. "Portfolio Choice with Illiquid Assets." *Management Science,* November 2014

Brunnermeier, Markus K. and Lasse Heje Pedersen. "Market Liquidity and Funding Liquidity." *Review of Financial Studies*, June 2009

Frazzini, Andrea, David Kabiller and Lasse Heje Pedersen. "Buffett's Alpha." *Financial Analysts Journal*, fourth-quarter 2018

Furnham, Adrian and Richard Rawles. "Sex Differences in the Estimation of Intelligence." *Journal of Social Behavior and Personality*, September 1995

Furnham, Adrian and Richard Rawles. "Correlations Between Self-Estimated and Psychometrically Measured IQ." *Journal of Social Psychology*, August 1999

Furnham, Adrian, Emma Reeves and Salima Budhani. "Parents Think Their Sons Are Brighter Than Their Daughters: Sex Differences in Parental Self-Estimations and Estimations of Their Children's Multiple Intelligences." *Journal of Genetic Psychology,* April 2002

8 We use a value for k of 1.05, which corresponds to a half-life of approximately 0.66 years.

Keynes, John Maynard. *The General Theory of Employment, Interest and Money.* London: Palgrave Macmillan, 1936

Longstaff, Francis A. "Valuing Thinly Traded Assets." *Management Science*, August 2018

Nadauld, Taylor D., Berk A. Sensoy, Keith Vorkink and Michael S. Weisbach. "The Liquidity Cost of Private Equity Investments: Evidence from Secondary Market Transactions." *Journal of Financial Economics*, December 2018. https://doi.org/10.1016/j.jfineco.2018.11.007

Reilly, Jacqueline and Gerry Mulhern. "Gender Difference in Self-Estimated IQ: The Need for Care in Interpreting Group Data." *Personality and Individual Differences*, February 1995

Van den Berg, Thijs. "Calibrating the Ornstein–Uhlenbeck (Vasicek) Model," May 2011. https://www.statisticshowto.datasciencecentral.com/wp-content/uploads/2016/01/Calibrating-the-Ornstein.pdf

Shimko, David C., "Finance in Continuous Time: A Primer." Kolb Publishing, 1992.

Vasicek, Oldrich. "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics*, November 1977

Zuckerman, Ezra W. and John T. Jost. "What Makes You Think You're So Popular? Self-Evaluation Maintenance and the Subjective Side of the 'Friendship Paradox.'" *Social Psychology Quarterly*, September 2001

APPENDIX: THE SOLUTION TO EQUATION 4

The OU process – Equation 1 in the body of this paper – is copied as follows:

$$d\alpha_t = k(\mu - \alpha_t)dt + \sigma dW_t \qquad \text{(A.1)}$$

where $k > 0$ is the mean-reverting speed, μ denotes the long-run mean and $\sigma > 0$ represents the volatility. The solution to (A.1) is

$$\alpha_t = e^{-kt}\alpha_0 + \mu(1 - e^{-kt}) + \sigma e^{-kt}\int_0^t e^{ks}dW_s. \qquad \text{(A.2)}$$

It is normal-distributed, with mean and variance as follows:

$$E(\alpha_t) = e^{-kt}\alpha_0 + \mu(1 - e^{-kt}) \qquad \text{(A.3)}$$

$$VAR(\alpha_t) = \frac{\sigma^2(1-e^{-2kt})}{2k}. \qquad \text{(A.4)}$$

The discounted liquidity premium for the continuous component is

$$E\left(\int_0^T e^{-rt}\alpha_t dt\right) = \int_0^T e^{-rt}\left(e^{-kt}\alpha_0 + \mu(1 - e^{-kt})\right)dt \qquad \text{(A.5)}$$

$$= \frac{\alpha_0 - \mu}{r+k}\left(1 - e^{-(r+k)T}\right) + \frac{\mu}{r}\left(1 - e^{-rT}\right).$$

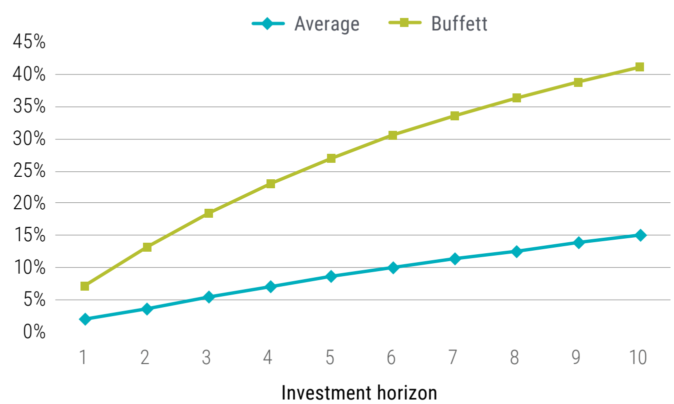
The jump component is

$$E\left(\int_0^T e^{-rt}J dq_t\right) = \int_0^T e^{-rt}\lambda J dt = \frac{\lambda J}{r}\left(1 - e^{-rT}\right). \qquad \text{(A.6)}$$

Combining Equation A.3 with Equation A.4, we obtain Equation 5 as

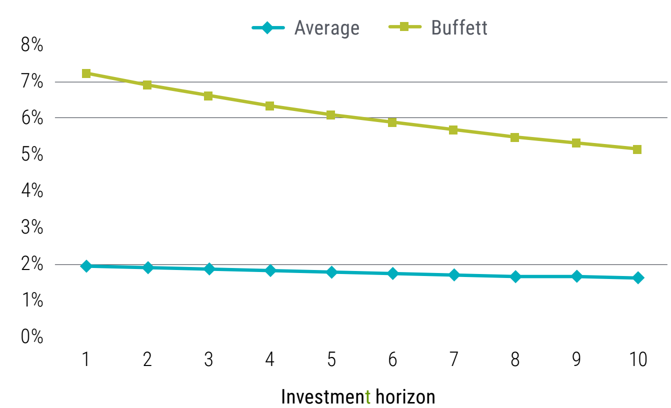
$$E\left(\int_0^T e^{-rt}J dq_t\right) = \int_0^T e^{-rt}\lambda J dt = \frac{\lambda J}{r}\left(1 - e^{-rT}\right). \qquad \text{(A.7)}$$

Exhibit 2a: Total illiquidity discounts



Source: PIMCO and Bloomberg as of 31 March 2018

Exhibit 2b: Annual illiquidity discounts



Source: PIMCO and Bloomberg as of 31 March 2018

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